Definite integral. Substitution in a definite integral. Integration by parts.
Some applications of definite integrals. Areas of plane regions

Suppose a function $f(x)$ is defined on an interval $[a;b]$. Let's divide the interval $[a;b]$ into n arbitrary parts by points $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$, choose an arbitrary point ξ_k on every elementary interval $[x_{k-1}; x_k]$ and calculate the length of each such interval: $\Delta x_k = x_k - x_{k-1}$. The **integral sum** for $f(x)$ on $[a;b]$ is the sum of the form $\sigma = \sum_{k=1}^{n} f(\xi_k)\,\Delta x_k$, and this sum has a finite limit I, if for every $\varepsilon > 0$ there is a such number $\delta > 0$, that at $\max \Delta x_k < \delta$ the inequality $|\sigma - I| < \varepsilon$ is true for any set of points ξ_k.

The definite integral of a function $f(x)$ on an interval $[a;b]$ (or within a and b) is the limit of the integral sum conditional on the fact, that the length of the largest elementary segment $(\max \Delta x_k)$ reaches zero:

$$I = \int_a^b f(x)\,dx = \lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(\xi_k)\,\Delta x_k \,.$$

Numbers a and b are called **the lower and the upper limits** of integration respectively.

Basic properties of a definite integral

1. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
2. $\int_a^a f(x)\,dx = 0$
3. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
4. $\int_a^b [f_1(x) \pm f_2(x)]\,dx = \int_a^b f_1(x)\,dx \pm \int_a^b f_2(x)\,dx$
5. $\int_a^b c\, f(x)\,dx = c\int_a^b f(x)\,dx$, where c is a constant
6. If $m \le f(x) \le M$ on $[a;b]$, then $m\,(b-a) < \int_a^b f(x)\,dx < M\,(b-a)$